DEFORMATIONS OF POSITIVE SCALAR CURVATURE METRICS

ALESSANDRO CARLOTTO

In these lectures I shall present a series of results concerning the interplay between two different curvature conditions, in the special case when these are given by pointwise inequalities on the scalar curvature of a manifold, and the mean curvature of its boundary. Such results lie at two conceptual levels: on the one hand at the level of *compatibility* (i.e. is it possible to simultaneously satisfy the bounds, and what are the resulting topological implications), on the other hand at the level of *moduli space structure* (i.e. what can one say about the homotopy type of the associated space of metrics, when not empty, quotiented by the diffeomorphism group of the background manifold).

In particular, after a broad contextualization I will focus on recent joint work with Chao Li (Princeton University), where we give a complete topological characterization of those compact 3-manifolds that support Riemannian metrics of positive scalar curvature and mean-convex boundary and, in any such case, we prove that the associated moduli space of metrics is path-connected. In the same article, we also show how our methods can be refined so to construct continuous paths of non-negative scalar curvature metrics with *minimal* boundary, and to obtain analogous conclusions in that context as well. In particular, note that we can derive the path-connectedness of asymptotically flat scalar flat Riemannian 3-manifolds with minimal boundary, which goes in the direction of understanding the space of vacuum black-hole solutions to the Einstein equations in General Relativity.

Our work relies on a combination of earlier fundamental contributions by Gromov-Lawson and Schoen-Yau, on the smoothing procedure designed by Miao to handle singular interfaces, and on the interplay of Perelman's Ricci flow with surgery and conformal deformation techniques introduced by Codá Marques in dealing with the closed case.